

電子計数技術とその応用

名城大学 工学部
電気電子工学科
児玉 哲司

1. 電子（電流）の計測
2. デジタル相関器
3. 時間分解電子顕微鏡法

電子

質量

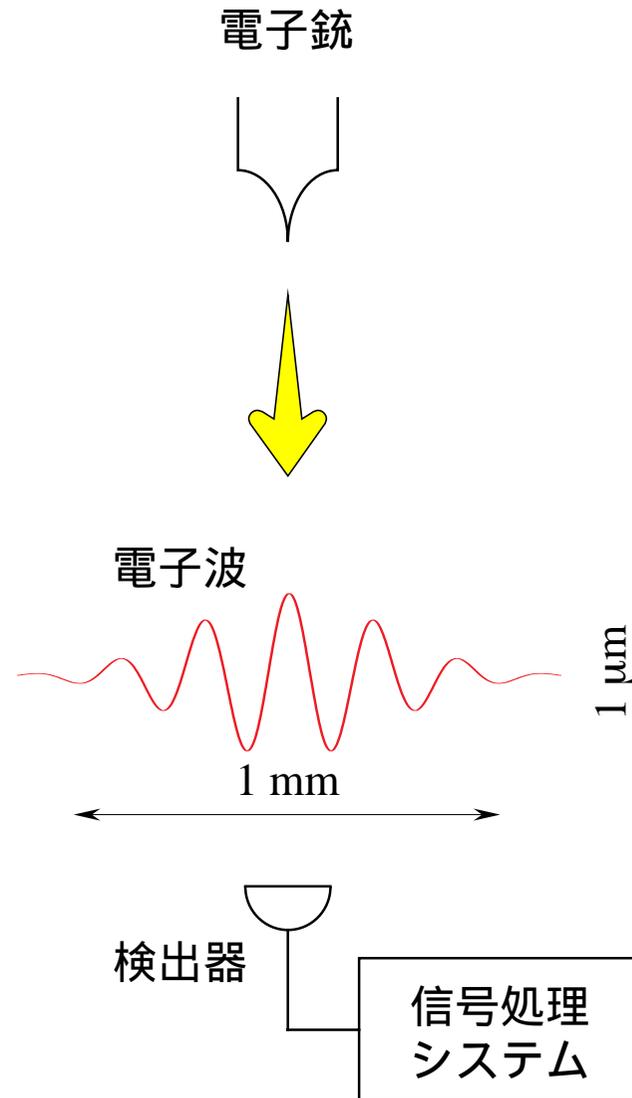
$$m=9.1 \times 10^{-31} \text{ kg}$$

電荷

$$e=1.6 \times 10^{-19} \text{ C}$$



数えることができる



計数システム

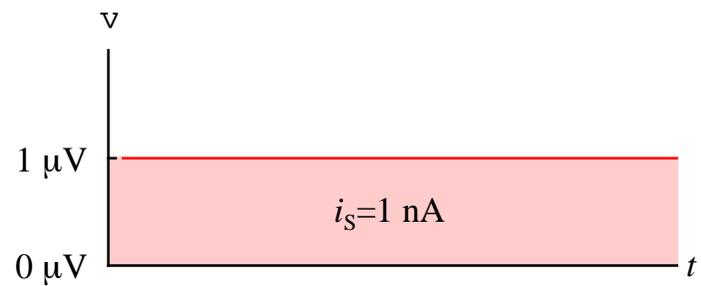
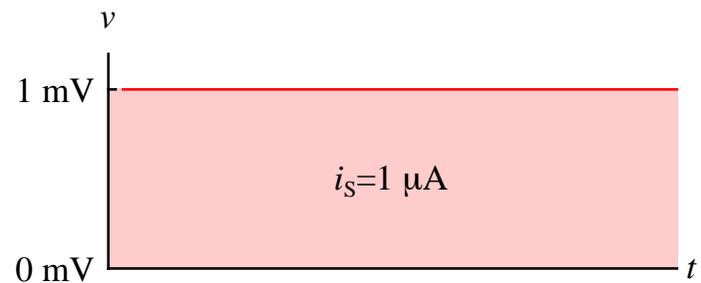
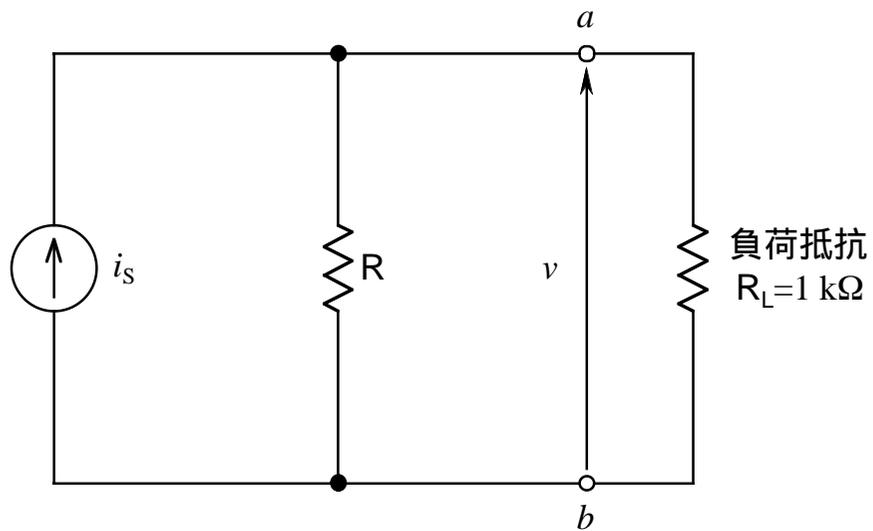
約10,000,000,000個の電子を数える必要があった。

もし1個に1秒かかると10,000,000,000秒 300年かかる。



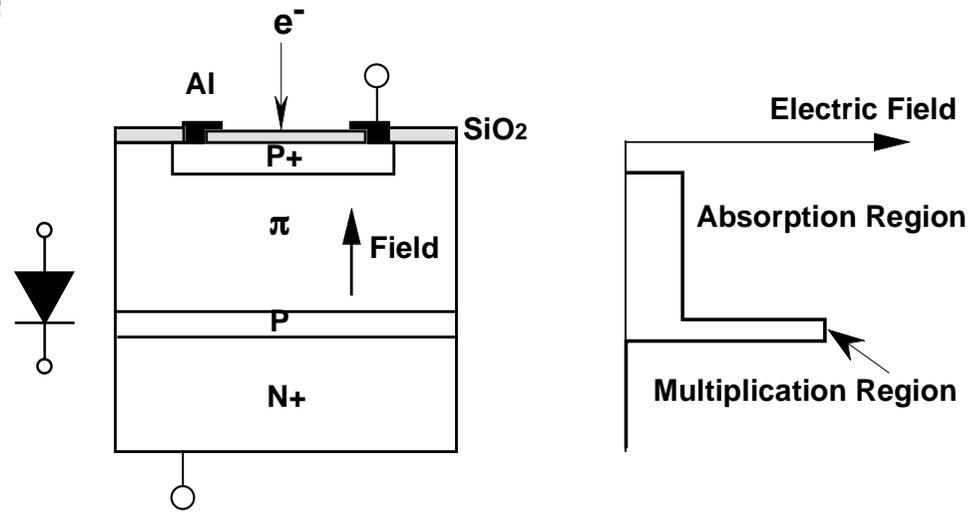
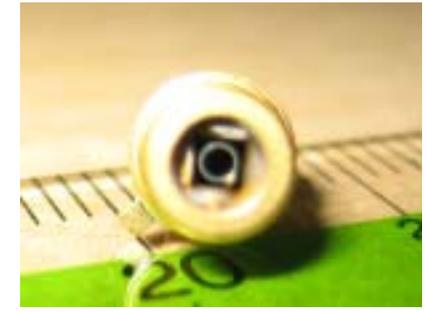
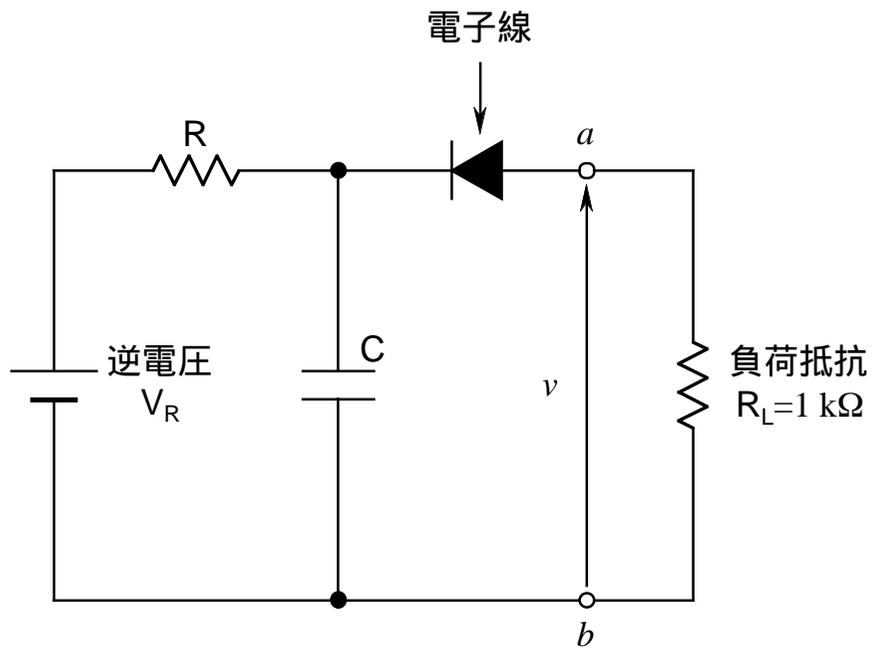
高速電子検出器
(アバランシェフォトダイオード)
超高速デジタル信号処理システム
(クロック5 GHzの電子回路)

電流の計測



電子電流の計測方法

フォトダイオード



電子電流の増幅

電子の運動エネルギー
50 keV

Siのイオン化エネルギー
3.6 eV



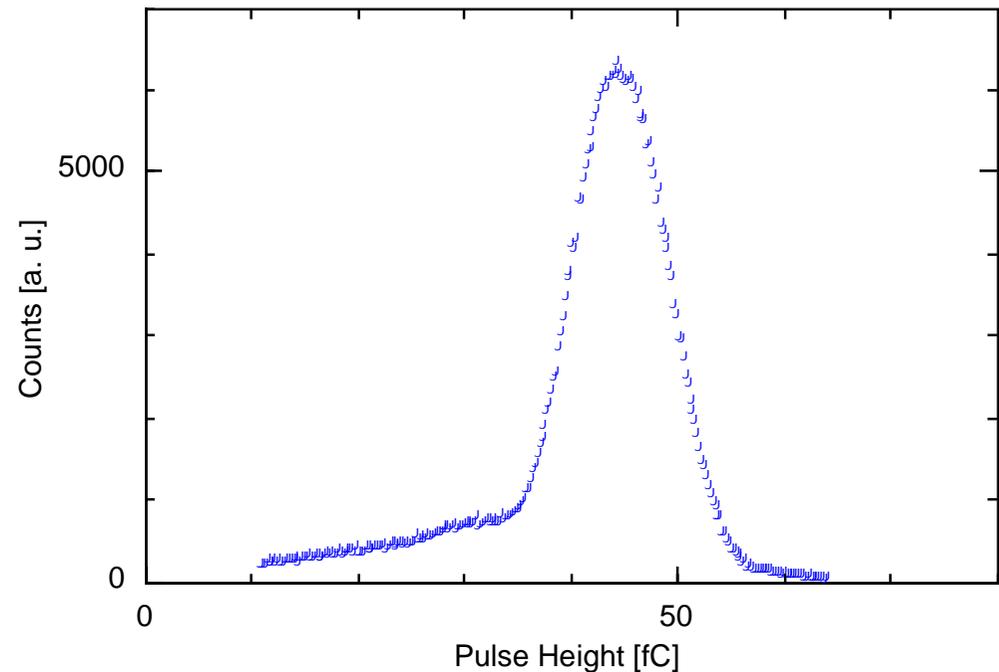
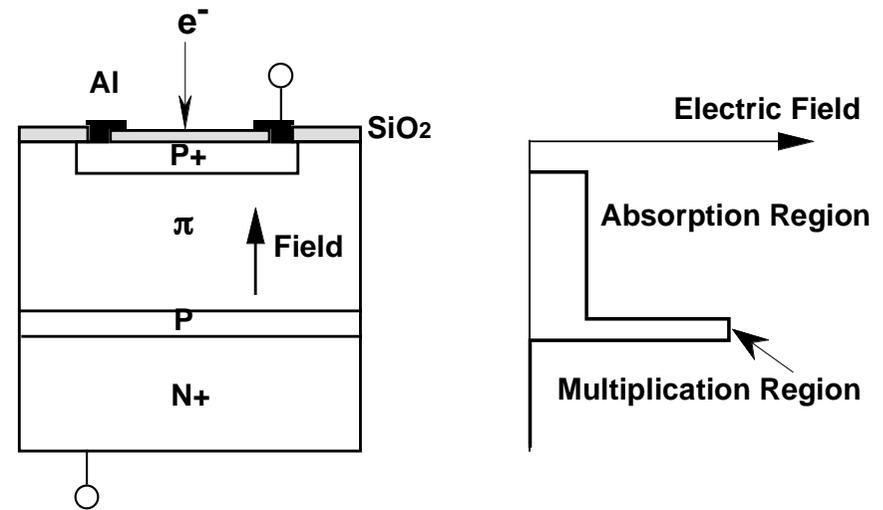
電子-正孔対
 $50 \text{ keV} / 3.6 \text{ eV}$
 $= 1.4 \times 10^4$

電気素量
 $e = 1.6 \times 10^{-19} \text{ C}$

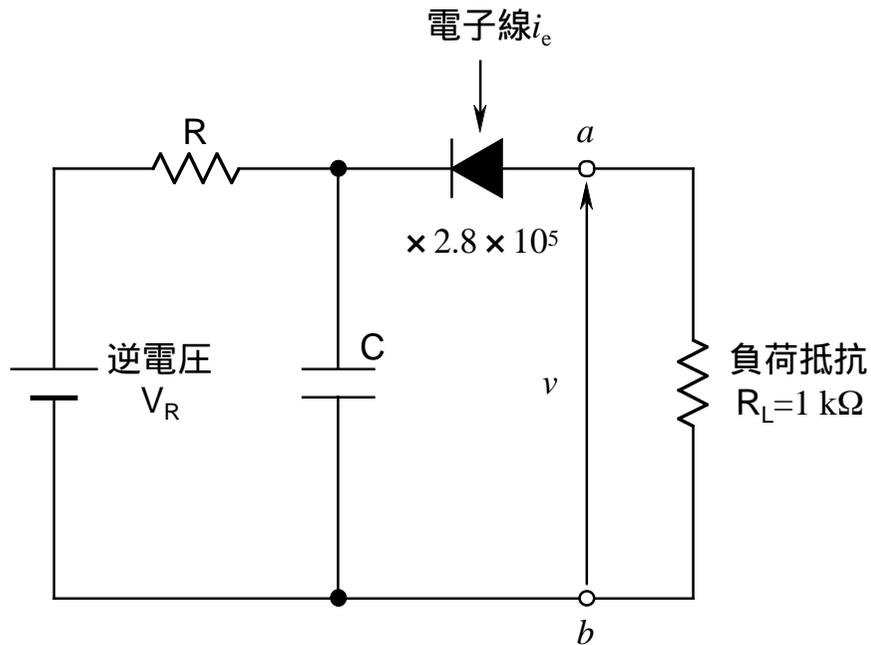
アバランシェ増倍
 $\times 20$



1 電子に対して
 $e \times 1.4 \times 10^4 \times 20$
 $= 45 \times 10^{-15} \text{ C}$



電子電流の計測

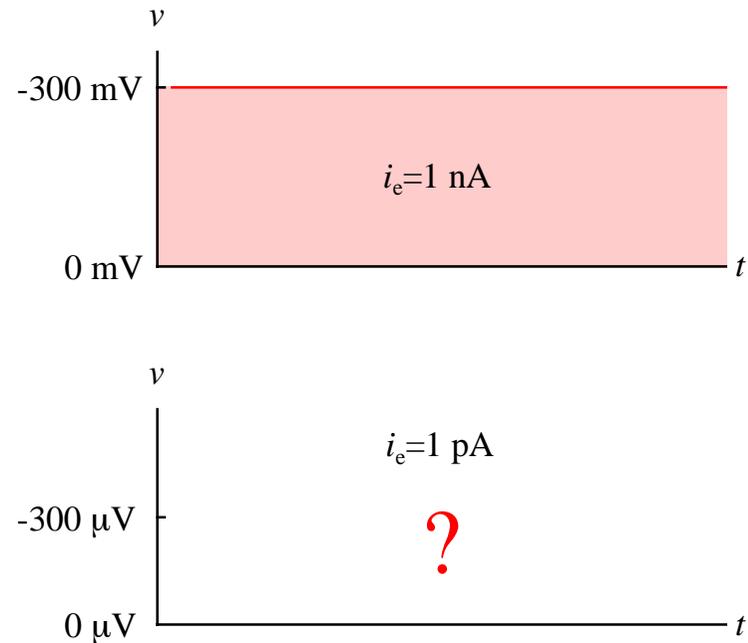


電気素量
 $e = 1.6 \times 10^{-19} \text{ C}$

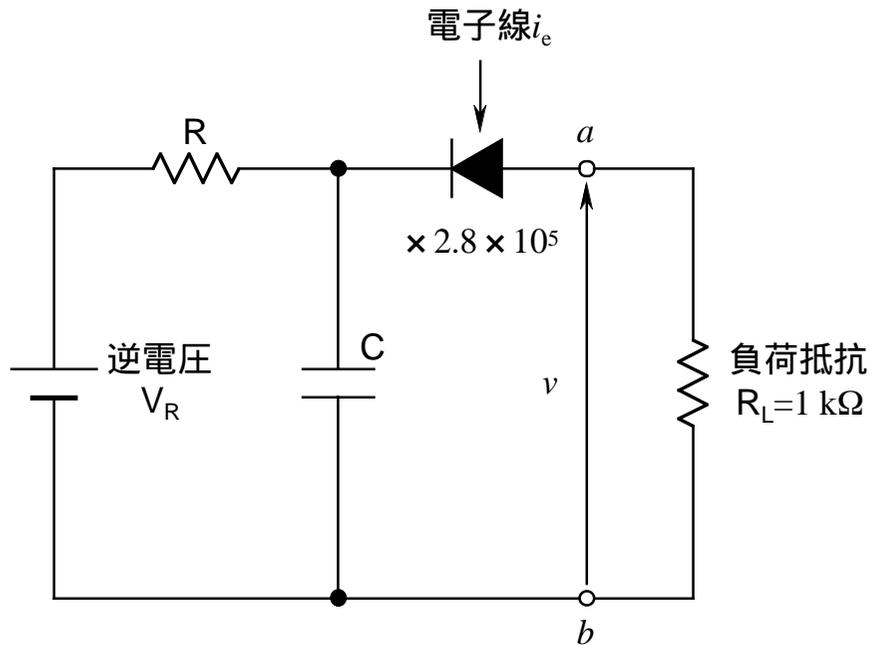
$$i_e = 1 \text{ pA}$$

$$= 10^{-12} \text{ A} / (1.6 \times 10^{-19} \text{ C})$$

$$= 6.3 \times 10^6 \text{ 1/s}$$



電子の計数

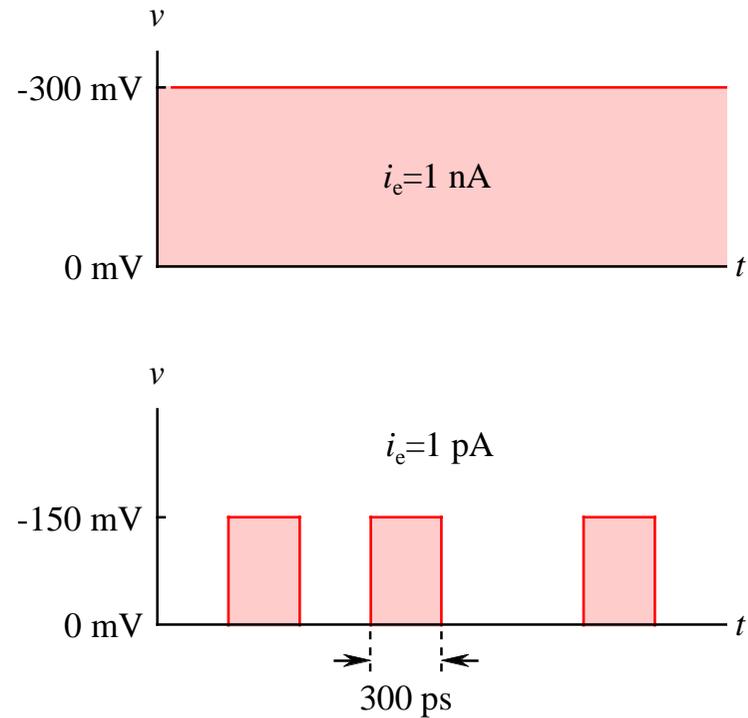


電気素量

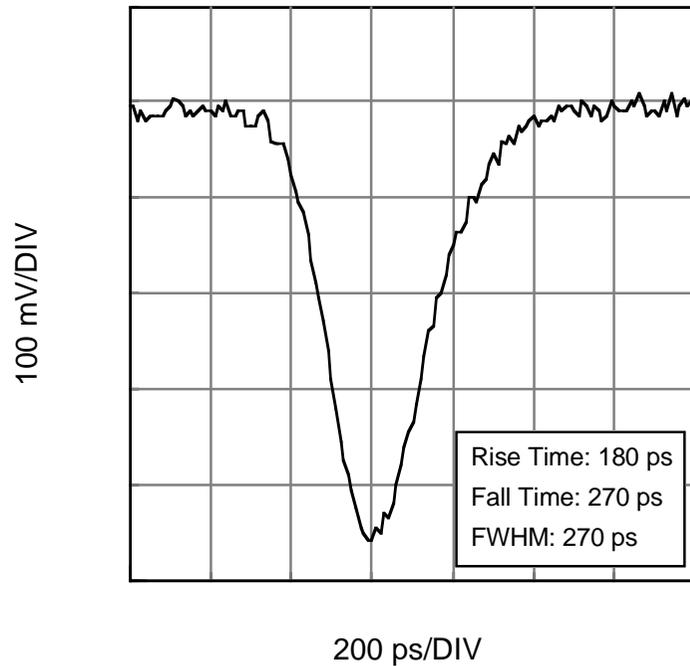
$$e = 1.6 \times 10^{-19} \text{ C}$$

パルス電流

$$\begin{aligned} i'_e &= e \times 2.8 \times 10^5 / 300 \text{ ps} \\ &= 45 \times 10^{-15} \text{ C} / (300 \times 10^{-12} \text{ s}) \\ &= 0.15 \times 10^{-3} \text{ A} \end{aligned}$$



1 電子に対する応答波形



アンプ

Anritsu A3H2121, DC-12 GHz, 20 dB

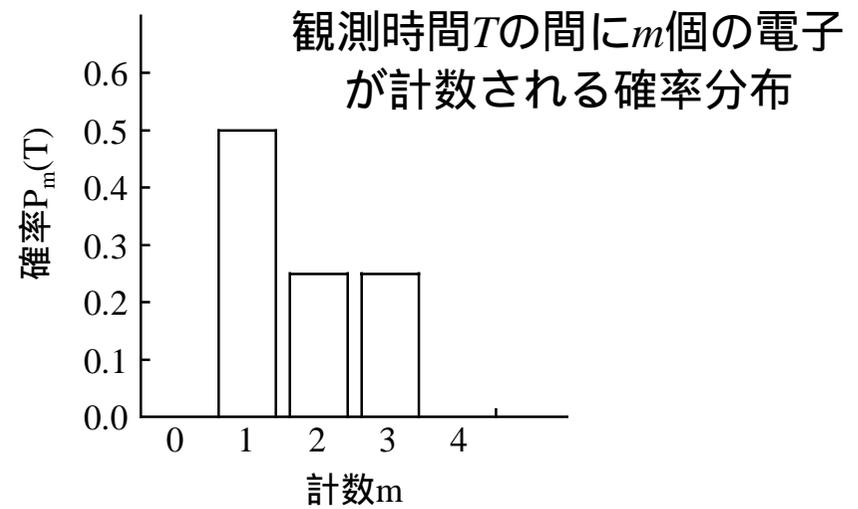
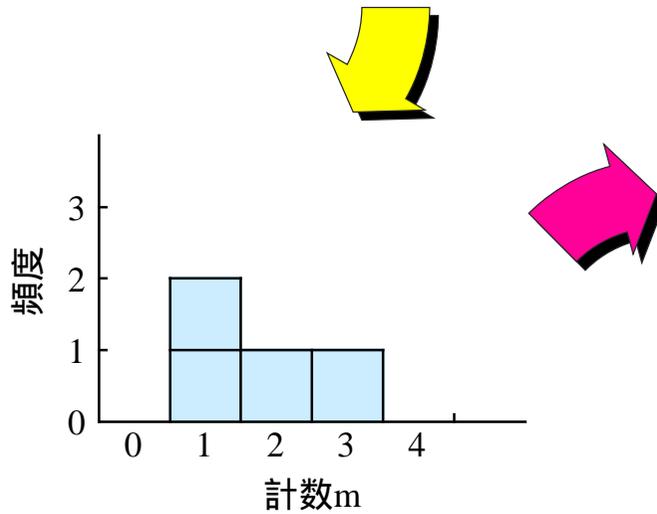
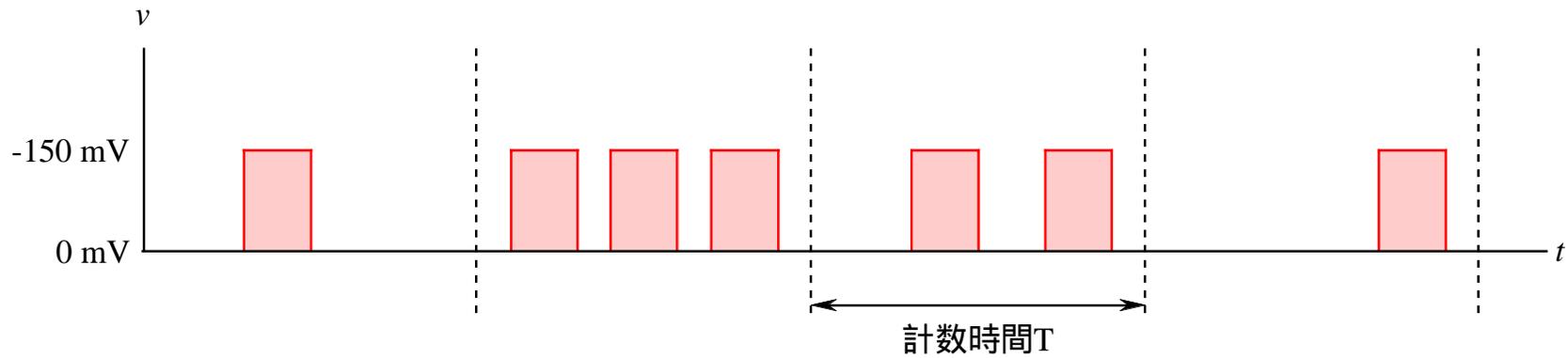
オシロスコープ

Tektronix SCD5000, DC-3.5 GHz

信号処理方法

1. 計数分布
2. 時間間隔の分布
3. 相関関数

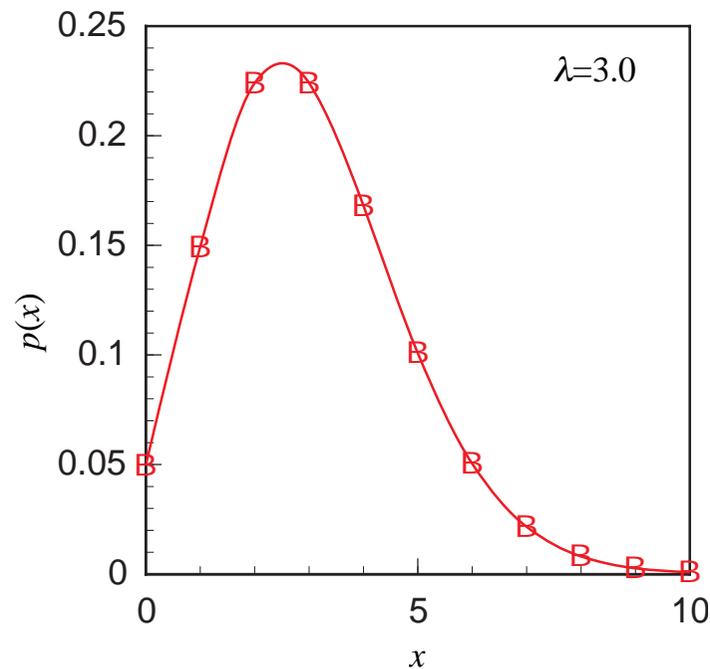
計数分布



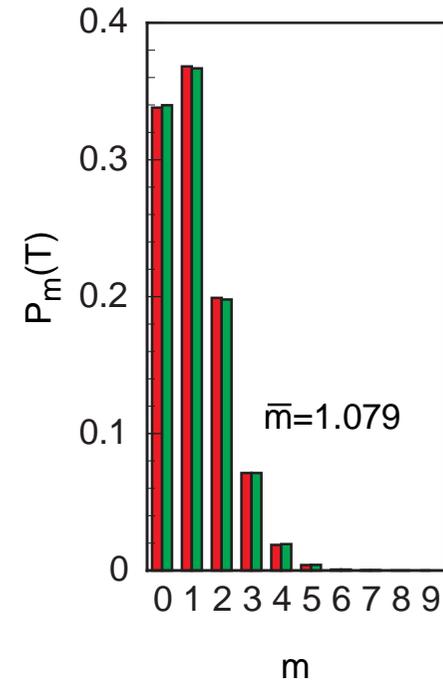
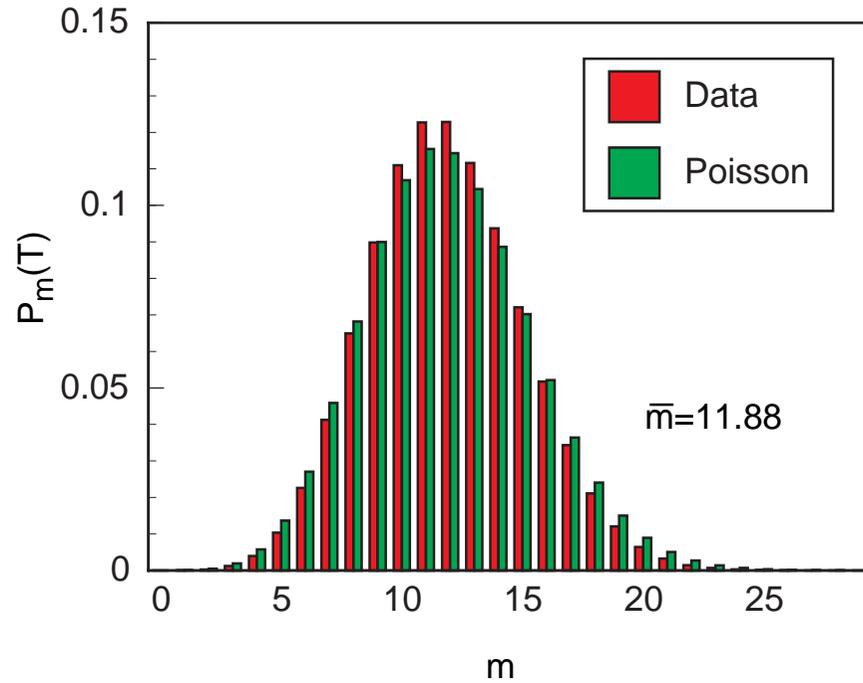
ポアソン分布

- ・非常に稀にしか起こらない事象を記述する確率分布
- ・ある時間あたりの稀現象の生起回数 x が確率変数
- ・ただ一つのパラメータ、平均 λ のみで特徴づけられる。

$$p(x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

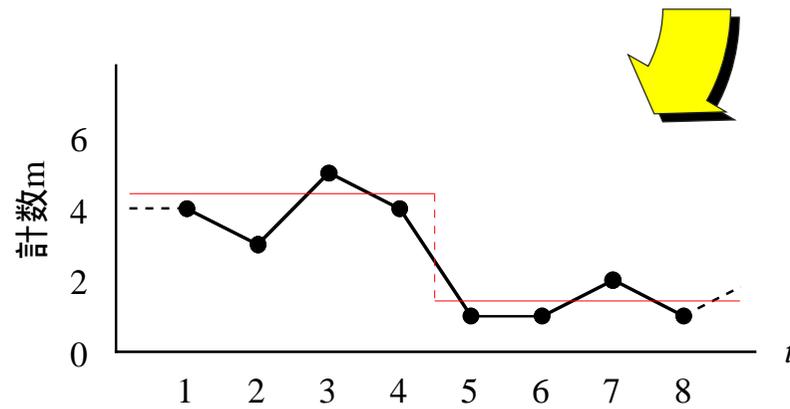
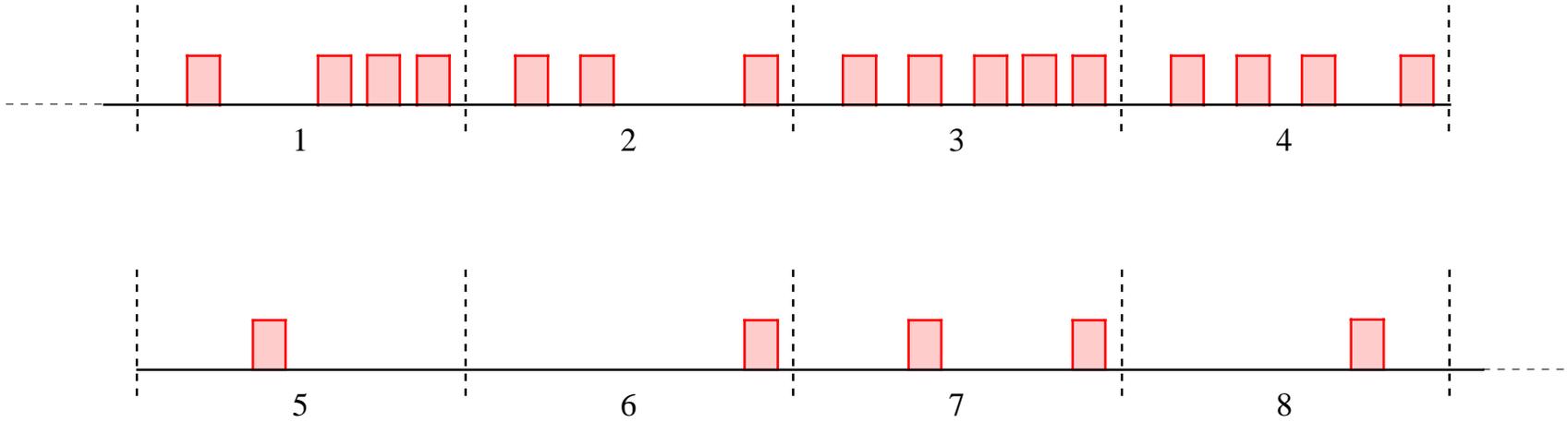


Experimental electron-count distribution

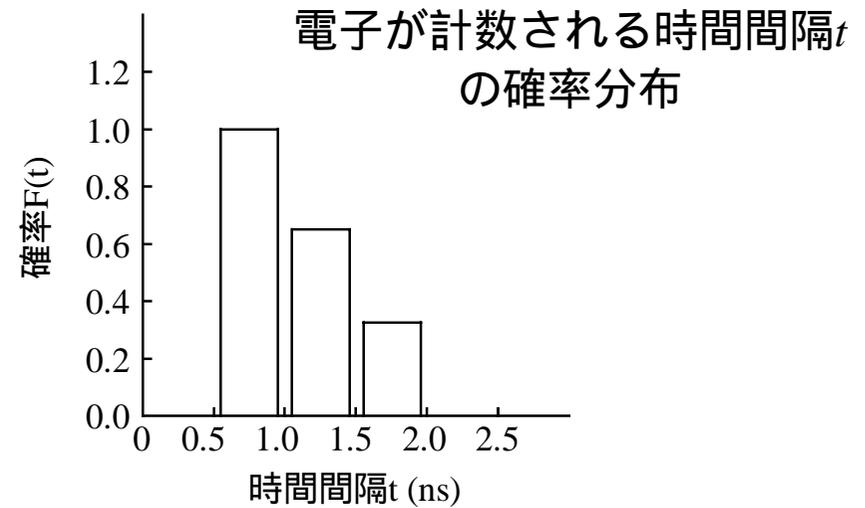
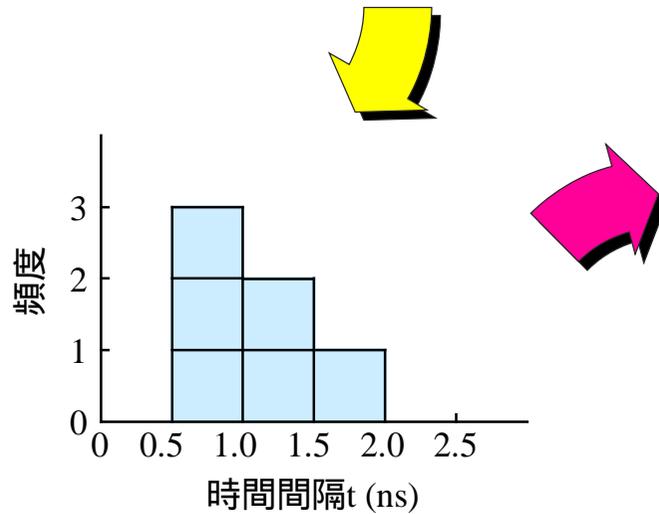
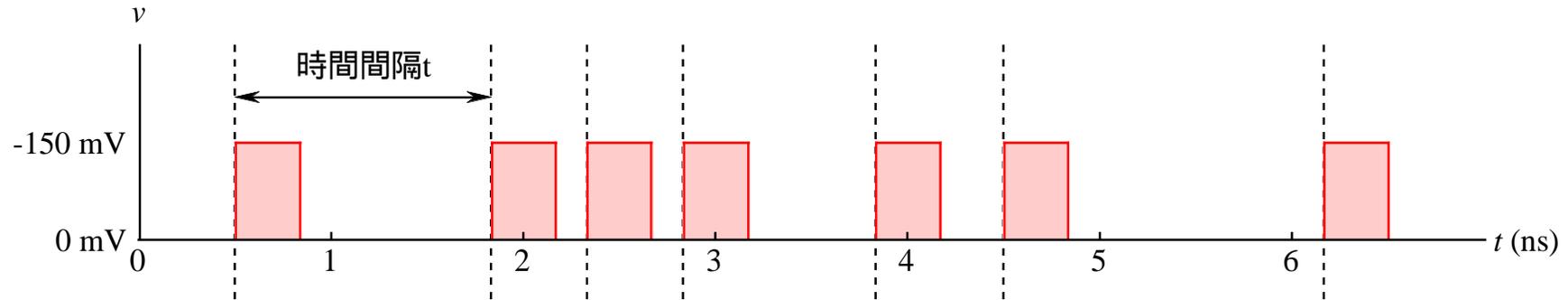


Counting time interval $T=85.33$ ns
Buffer memory 10^6 (8 bits) data points
Sampling rate 2.3×10^6 samples/s

時系列データ



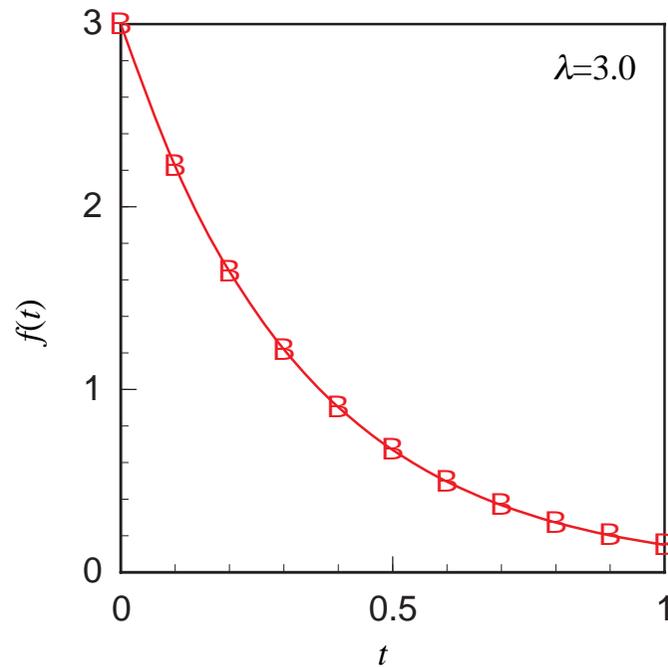
時間間隔の分布



指数分布

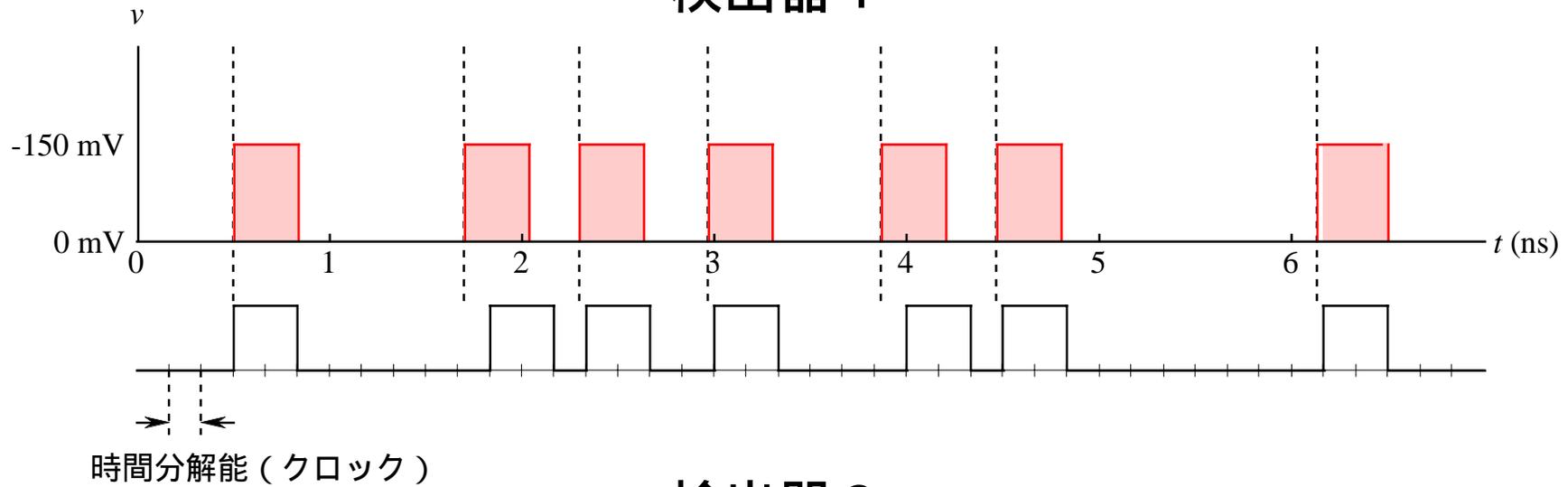
- ・ ランダム到着を記述する確率分布
- ・ 到着時間間隔 t が連続的な確率変数
- ・ パラメータ λ の逆数は時間間隔の平均値に等しい。

$$f(x) = \lambda e^{-\lambda t}$$

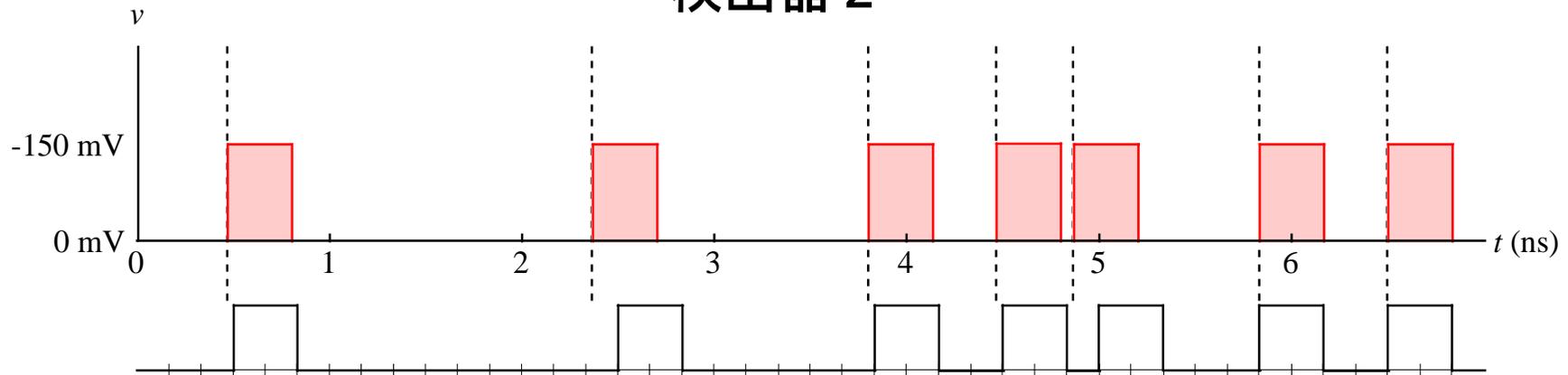


相関関数

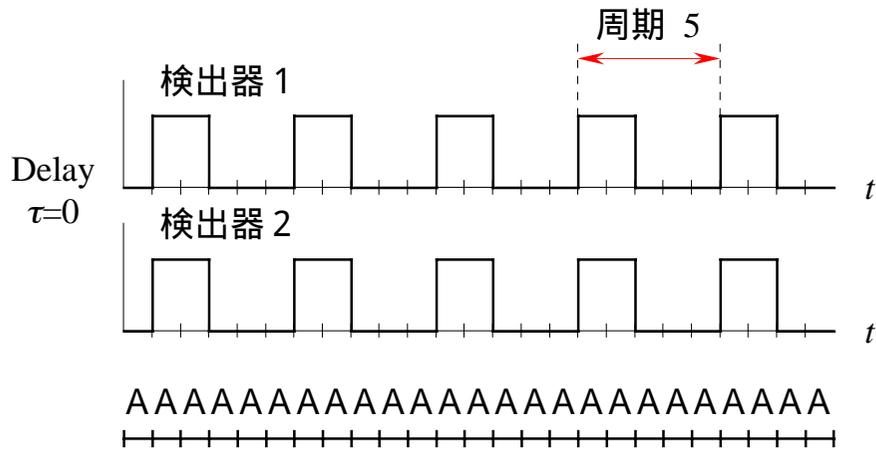
検出器 1



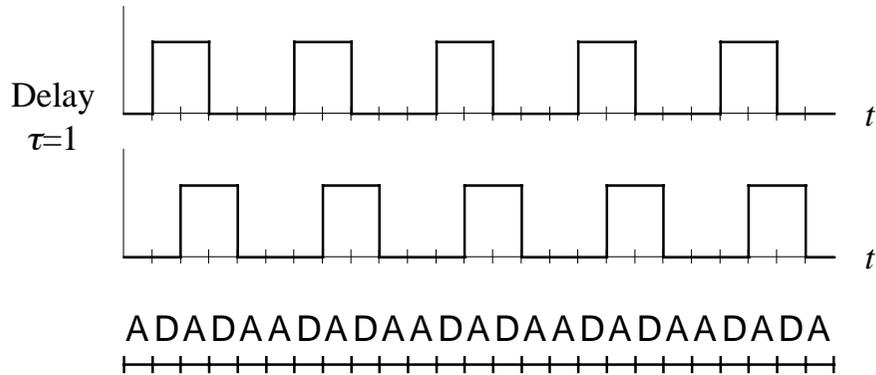
検出器 2



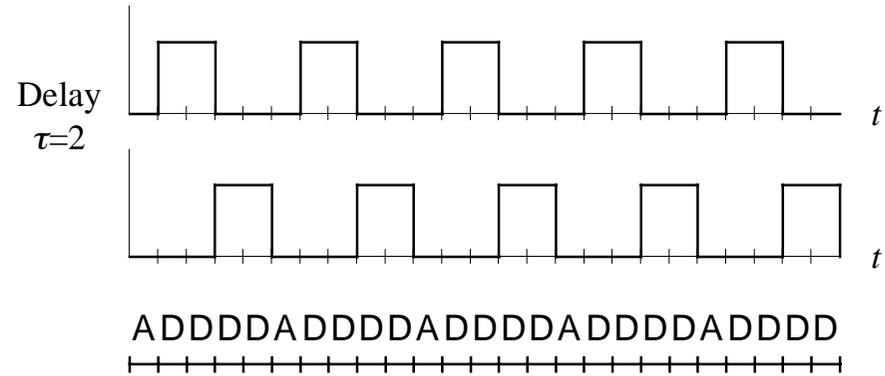
相関関数の演算



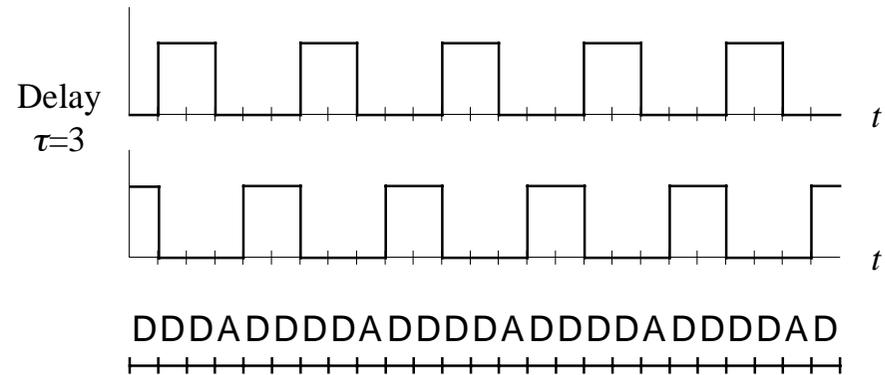
A 25, D 0 \rightsquigarrow $(25-0)/25=1.0$



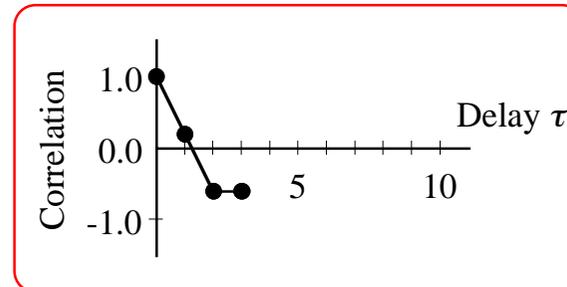
A 15, D 10 \rightsquigarrow $(15-10)/25=0.2$



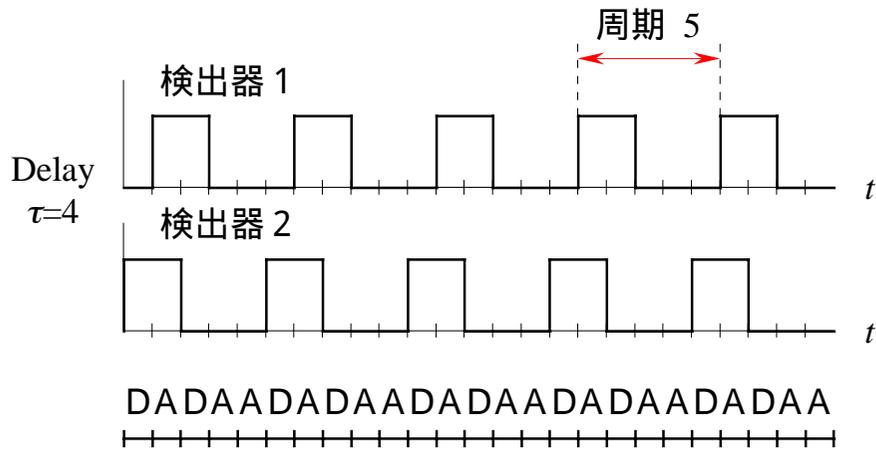
A 5, D 20 \rightsquigarrow $(5-20)/25=-0.6$



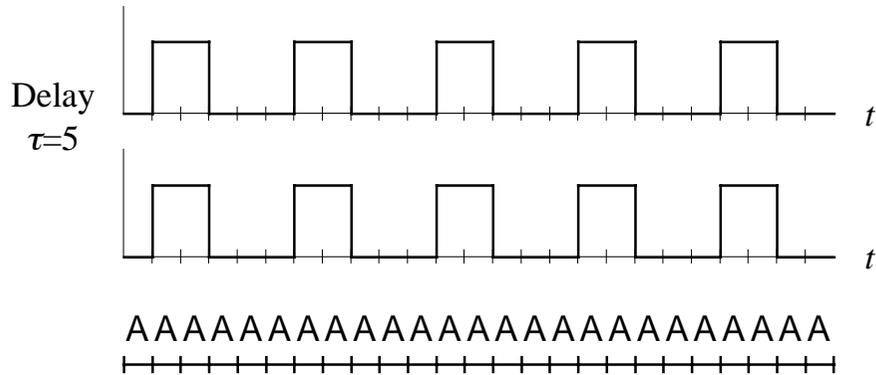
A 5, D 20 \rightsquigarrow $(5-20)/25=-0.6$



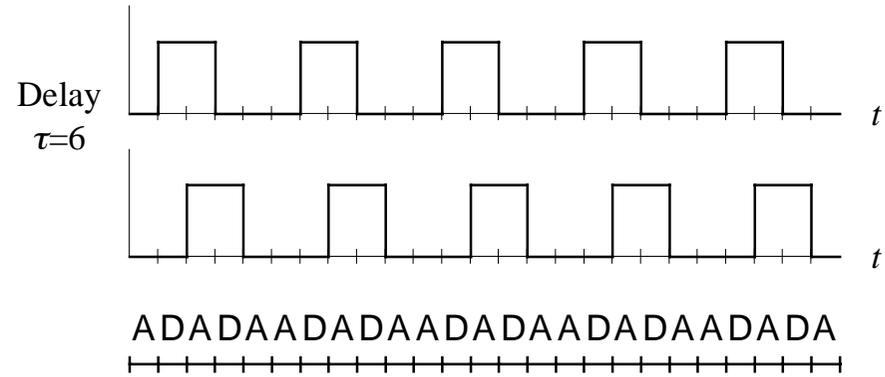
相関関数の演算



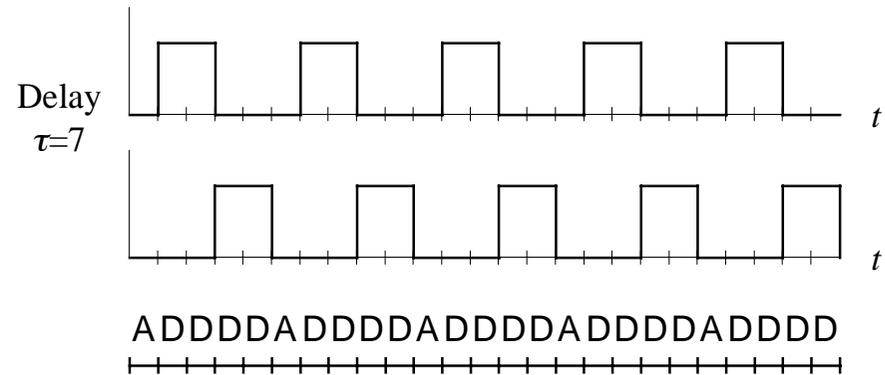
A 15, D 10 $\rightarrow (15-10)/25=0.2$



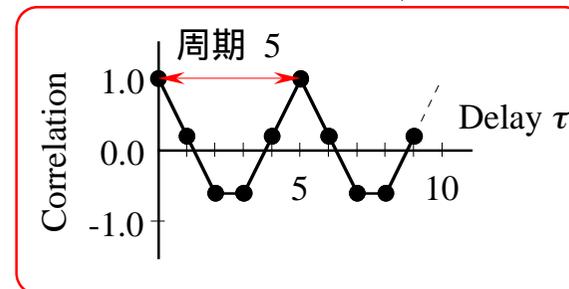
A 25, D 0 $\rightarrow (25-0)/25=1.0$



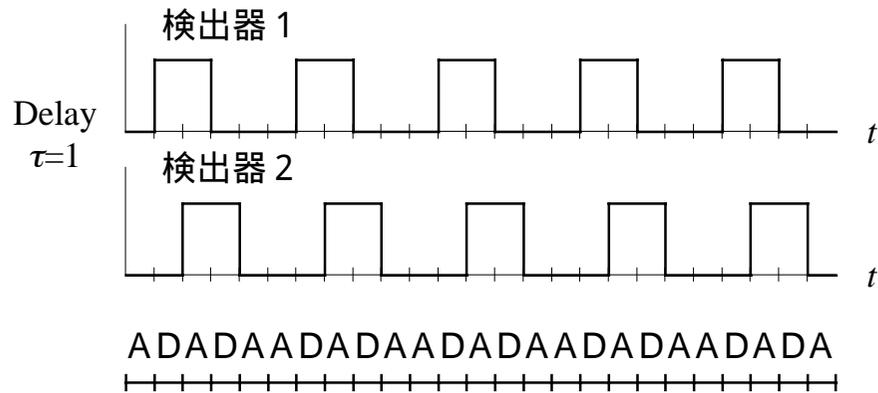
A 15, D 10 $\rightarrow (15-10)/25=0.2$



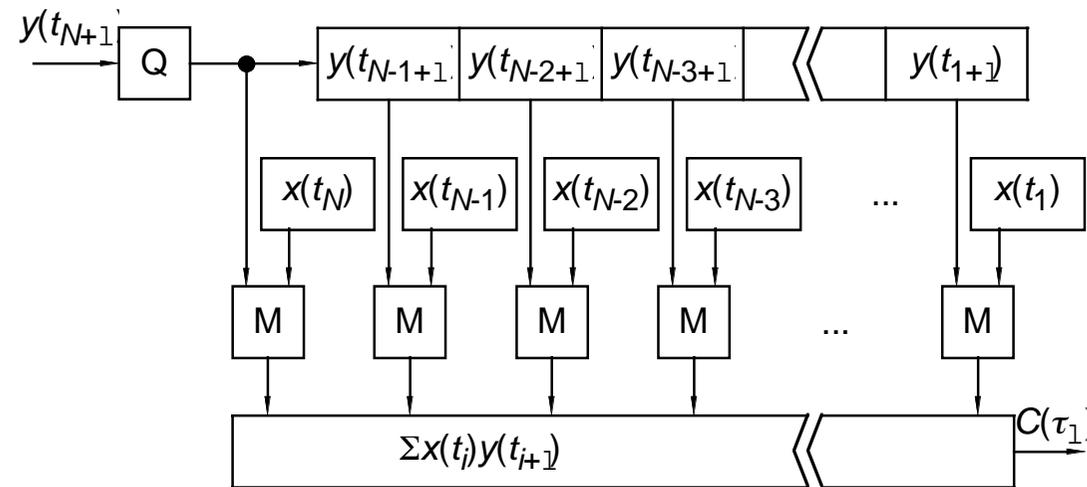
A 5, D 20 $\rightarrow (5-20)/25=-0.6$



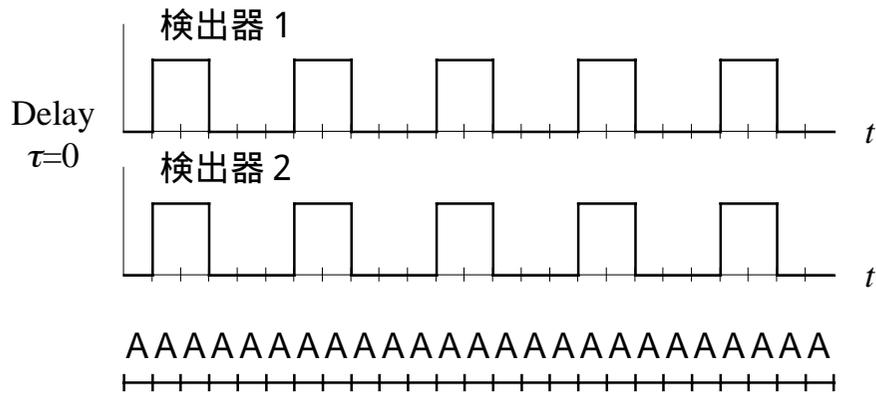
相関器



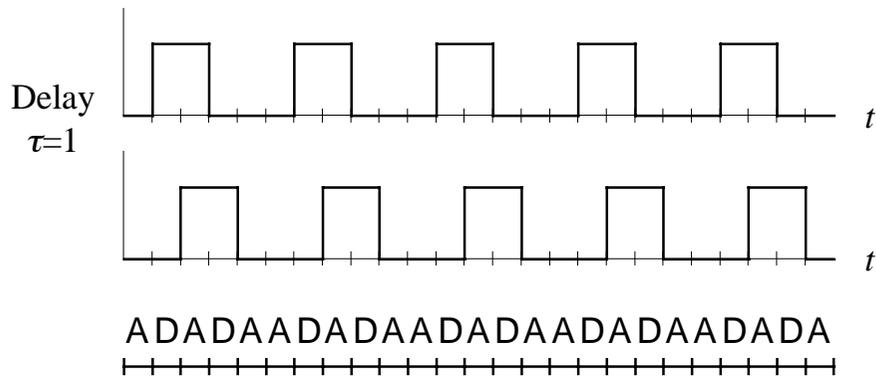
A 15, D 10 \rightarrow (15-10)/25=0.2



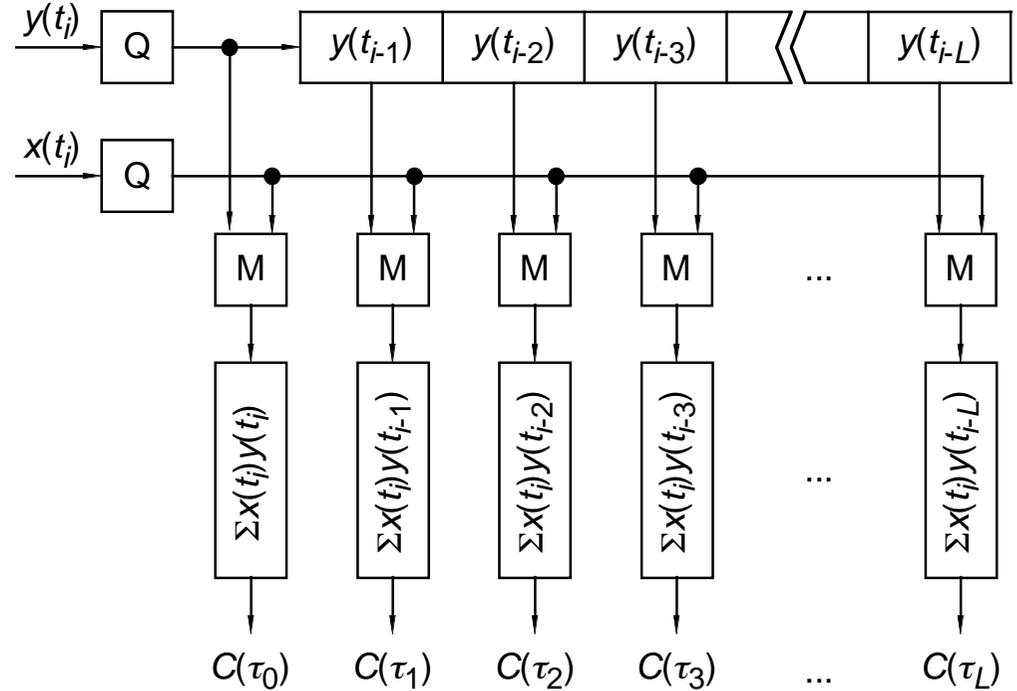
相関器



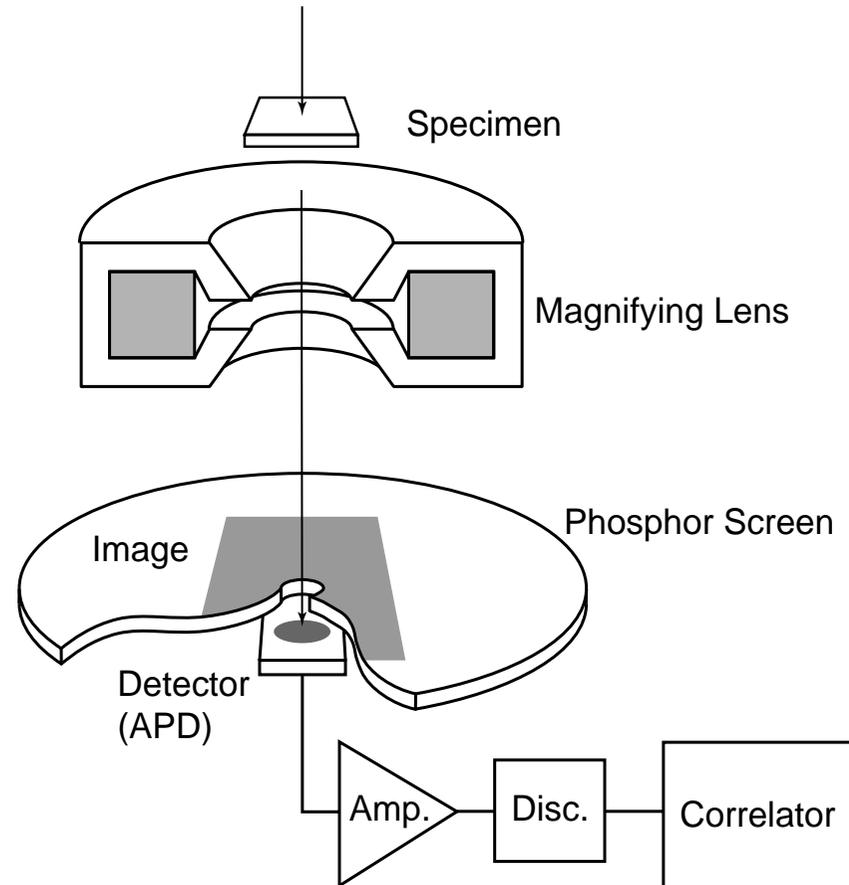
A 25, D 0 \rightarrow $(25-0)/25=1.0$



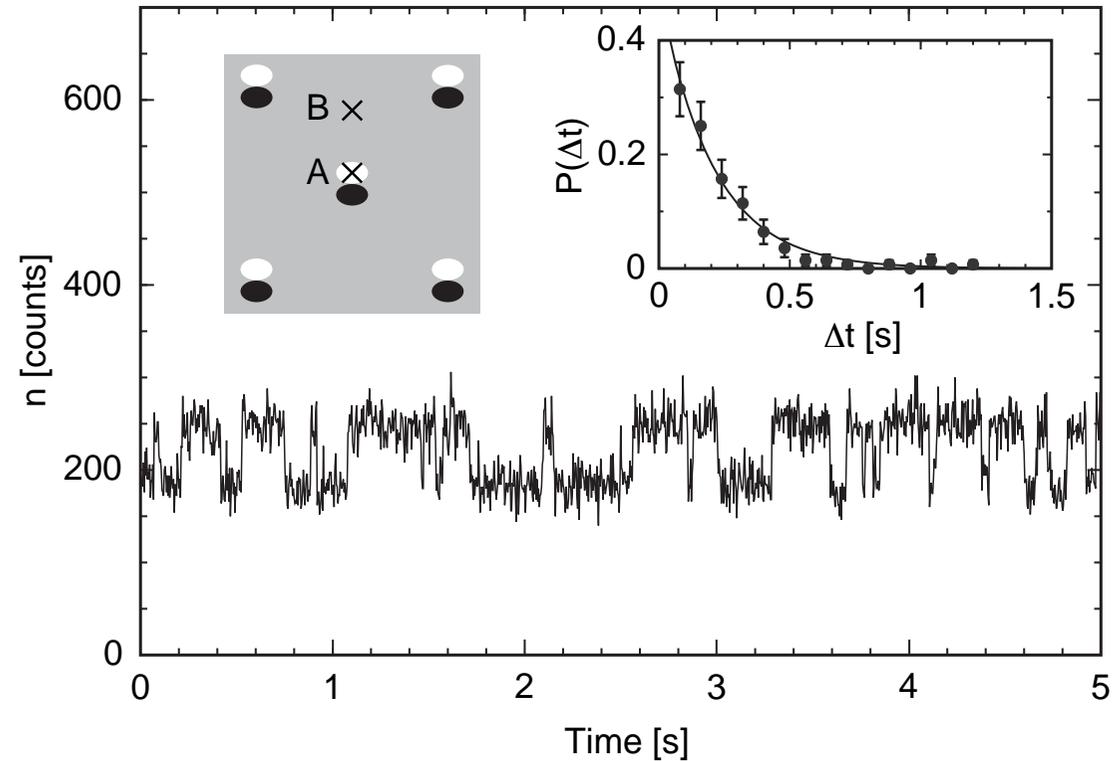
A 15, D 10 \rightarrow $(15-10)/25=0.2$



時間分解電子顯微鏡法

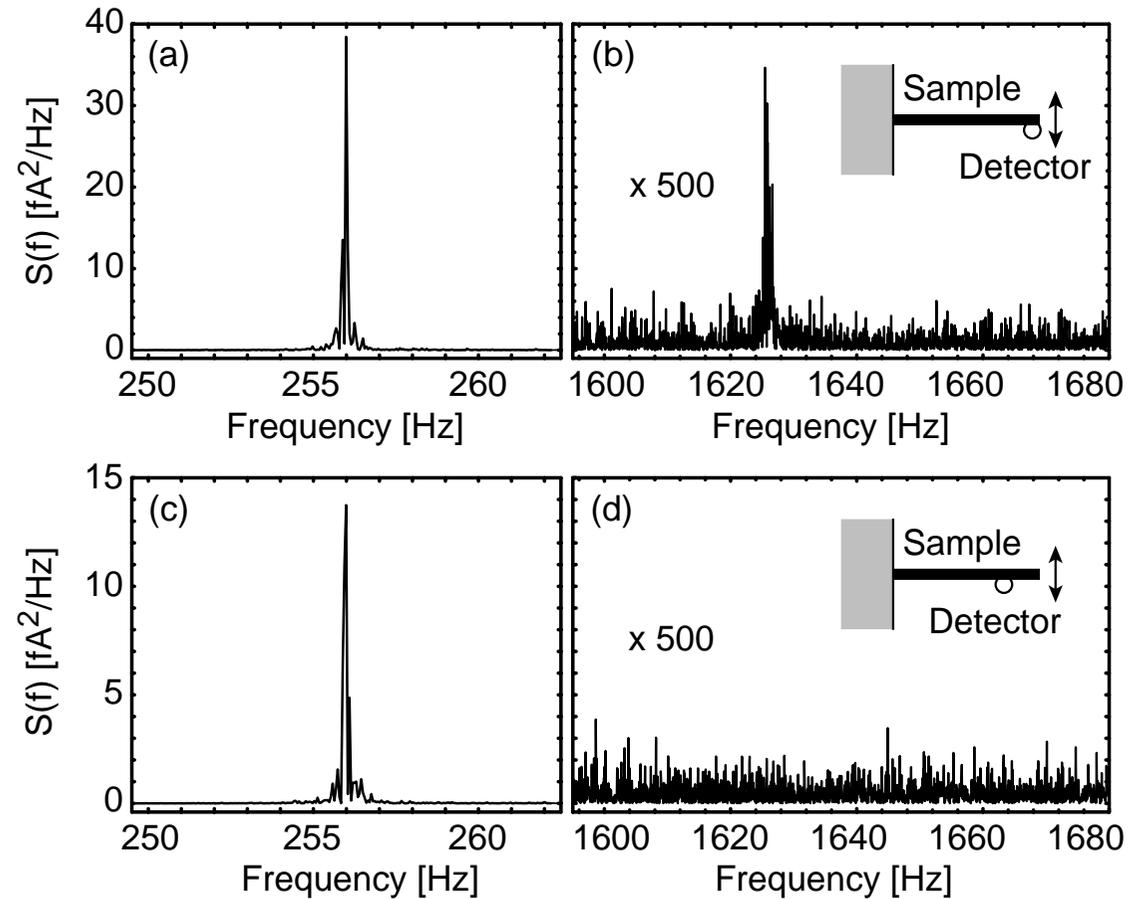


Application 1



Time-dependent image current showing vortex hopping in a type-II superconductor.

Application 2

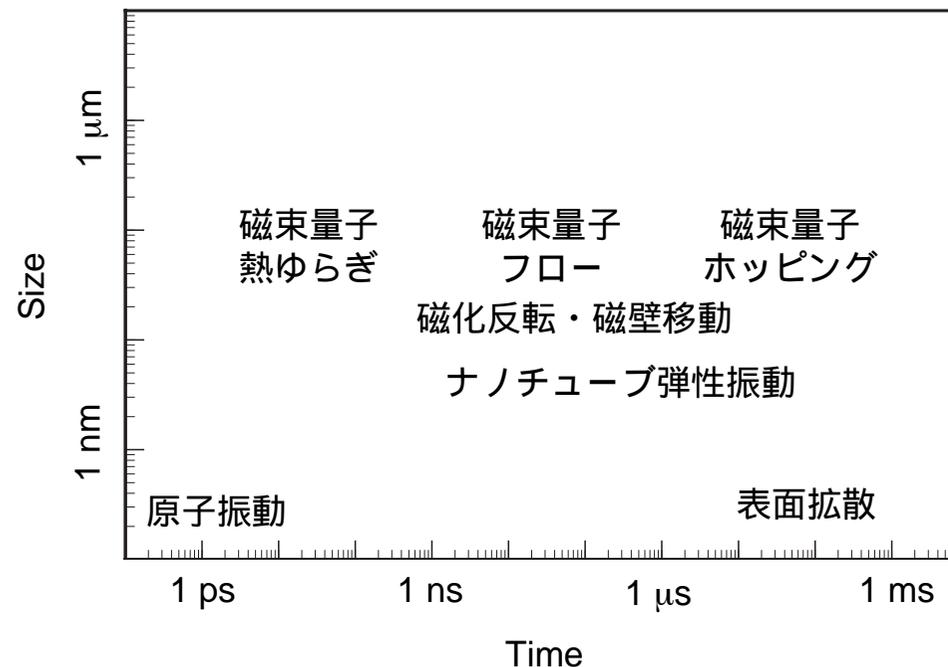


Power spectral densities of electron beam currents monitoring thermally driven oscillations of a Pt thin wire.

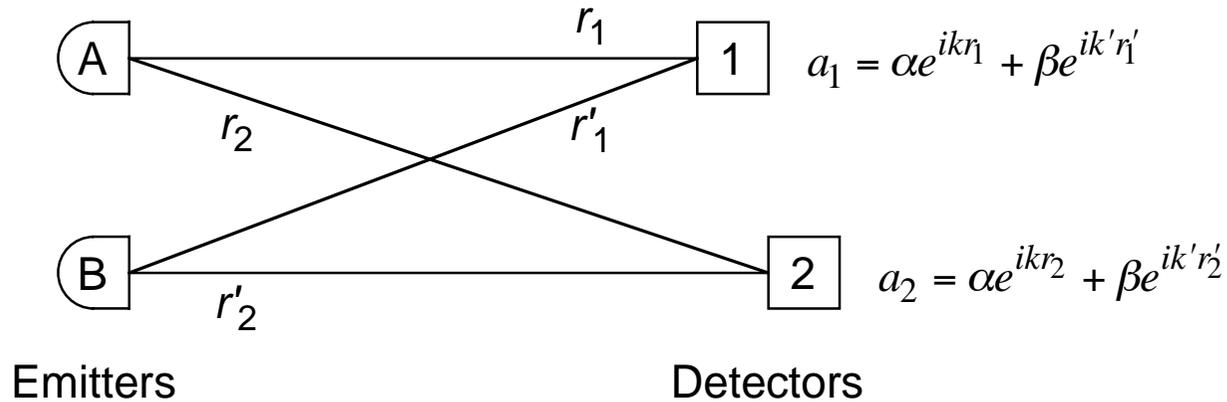
研究目標

電子顕微鏡は、0.1 nmの空間分解能で原子構造を決定できる重要な研究手段であるが、ダイナミクスに関しては、約10 msの現象までしか観察できなかった。

サブナノメートルの空間分解能で10 nsオーダーの現象を観測できる時間分解電子顕微鏡法を用いて、ナノチューブのようなハンドリングが困難なナノ材料の物性の評価方法を確立する。



Hanbury-Brown and Twiss experiment



$$\bar{I}_1 = |\alpha|^2 + |\beta|^2$$

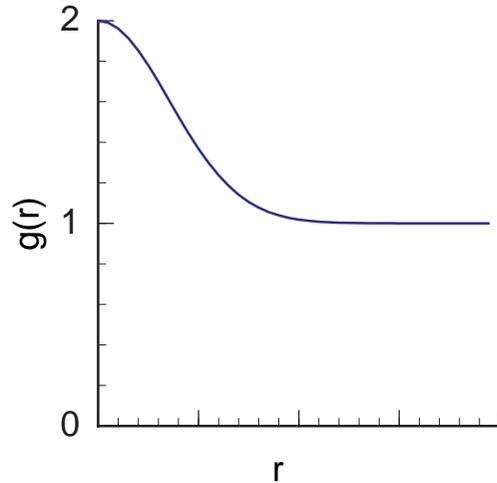
$$\bar{I}_2 = |\alpha|^2 + |\beta|^2$$

$$\overline{I_1 I_2} = \bar{I}_1 \bar{I}_2 + 2|\alpha|^2 |\beta|^2 \cos[k'(r'_1 - r'_2) - k(r_1 - r_2)]$$

Try to imagine the results of the Hanbury-Brown and Twiss experiment if it were performed with a beam of electrons

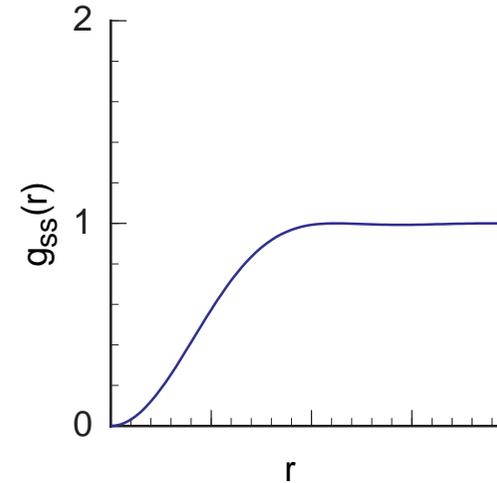
Pair correlation function

Noninteracting spin zero bosons



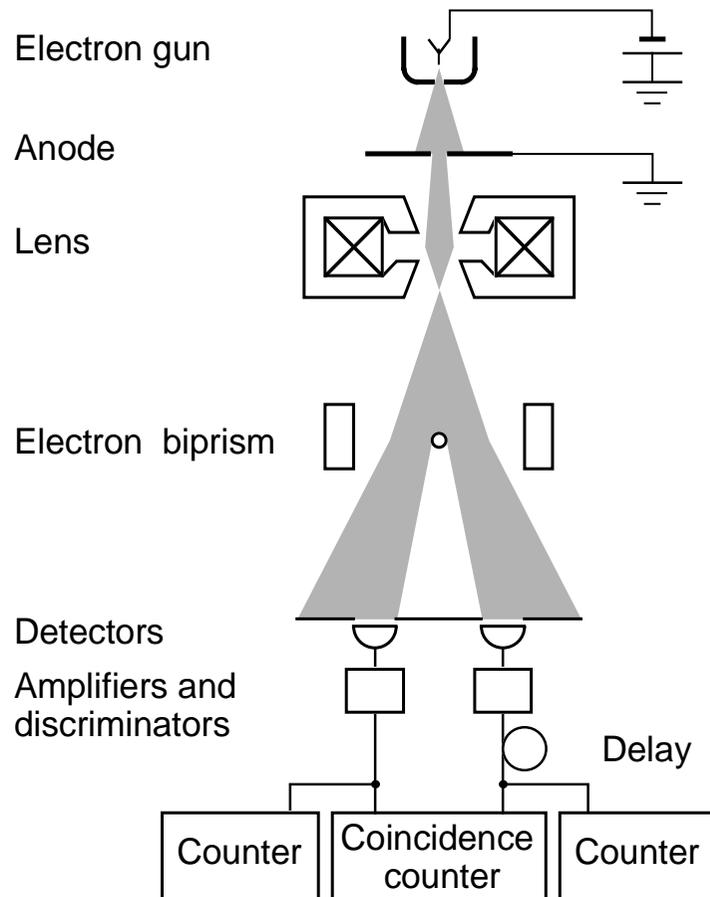
$$\begin{aligned} \langle \Phi | \psi^\dagger(\mathbf{r}) \psi^\dagger(\mathbf{r}') \psi(\mathbf{r}') \psi(\mathbf{r}) | \Phi \rangle &= n^2 g(\mathbf{r} - \mathbf{r}') \\ &= n^2 + \left| \int \frac{d^3 p}{(2\pi)^3} n_{\mathbf{p}} e^{-i\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')} \right|^2 \\ &= n^2 \left(1 + e^{-(\mathbf{r} - \mathbf{r}')^2 / 2\alpha^2} \right), \quad n_{\mathbf{p}} = c e^{-\alpha^2 (\mathbf{p} - \mathbf{p}_0)^2 / 2} \end{aligned}$$

Parallel spin, Noninteracting spin 1/2 fermions



$$\begin{aligned} \langle \Phi_0 | \psi_s^\dagger(\mathbf{r}) \psi_s^\dagger(\mathbf{r}') \psi_s(\mathbf{r}') \psi_s(\mathbf{r}) | \Phi_0 \rangle &= \left(\frac{n}{2} \right)^2 g_{ss}(\mathbf{r} - \mathbf{r}') \\ &= \left(\frac{n}{2} \right)^2 - \left| \int_0^{p_f} \frac{d^3 p}{(2\pi)^3} e^{-i\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')} \right|^2 \\ &= \left(\frac{n}{2} \right)^2 \left[1 - \frac{9}{x^6} (\sin x - x \cos x)^2 \right], \quad x = p_f |\mathbf{r} - \mathbf{r}'| \end{aligned}$$

Experimental setup for electrons



Field emission electron beam

Brightness $B/E=5 \times 10^3 \text{ A/cm}^2 \text{ eV}$

Energy spread (FWHM) 0.4 eV

Coherence time $\tau_c=3.9 \text{ fs}$

Transversal coherence length $l_t=0.27 \text{ mm}$

Average count $\bar{n}=0.0086$

Coincidence counter

Coincidence time window $T=200 \text{ ps}$

Time jitter $\sigma_t=20 \text{ ps}$

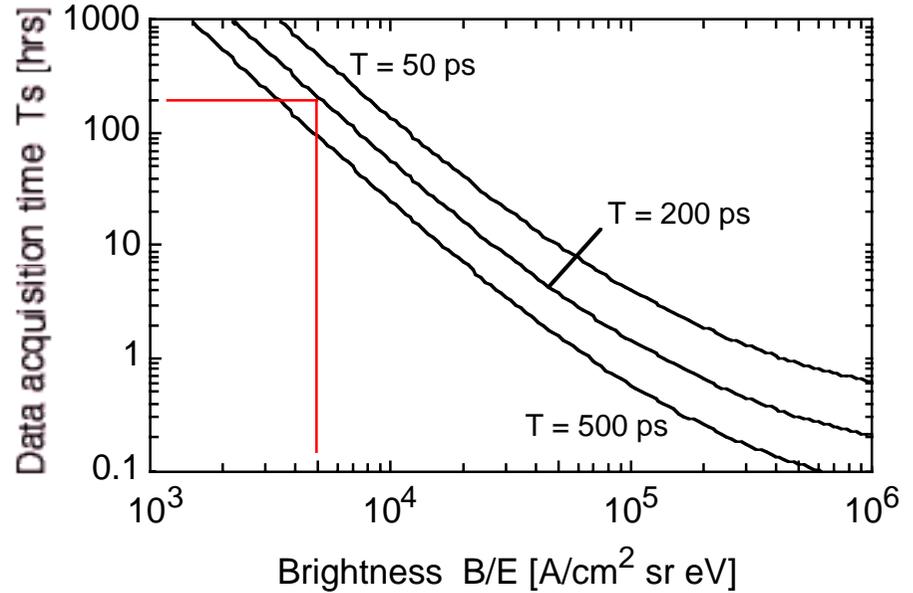
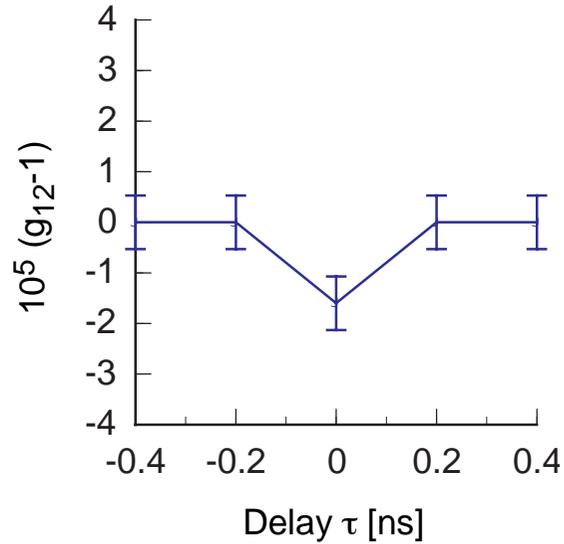
Avalanche photodiode

Sensitive area $a_y=0.4 \text{ mm}, a_z=1.2 \text{ mm}$

Detection efficiency $\zeta=0.85$

Dead time $\tau_d=480 \text{ ps}$

Estimate of data acquisition time



$$\tau_c \sim 4 \text{ (fs)}$$

$$|g_{12} - 1| \sim \frac{1}{2} \cdot \frac{4 \text{ (fs)}}{200 \text{ (ps)}} = 1 \times 10^{-5}$$